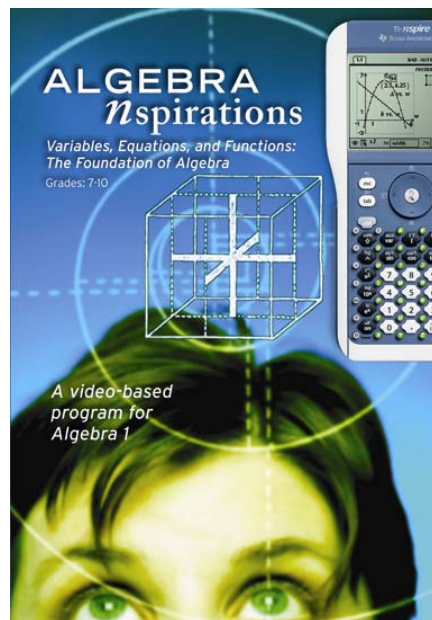




ALGEBRA NSPIRATIONS

Inequalities



Teacher's Guide

Series Overview

For Algebra 1 teachers looking for a video resource that uses graphing calculators, *Algebra Nspirations* provides an ideal solution. Each program in this series focuses on a key topic in algebra and uses real-world examples to explore these topics. In addition, all the relevant calculator keystrokes for the TI-Nspire calculator are provided. In addition, Math Labs allow for hands-on exploration of these topics.

Program Overview

Used in just about any industry, inequalities, like equations, are fundamental building blocks of algebra. Written and hosted by internationally acclaimed mathematics educator Dr. Monica Neagoy, this video explores inequalities—concepts, properties, solutions and notations—connects them to real-world contexts, and uses the TI-Nspire to make the algebra meaningful.

Learning Objectives

As a result of this program students will be able to do the following:

- Write and solve multistep inequalities
- Graph inequalities in one and two variables
- Solve problems that involve inequalities

Concepts explored: Variables, equations, functions, formulas, linear functions and equations, quadratic functions and equations, solving equations graphically

Grades: 7-12+

Investigation 1

Investigation 1 lays the groundwork for the concept of linear inequalities. Students should be comfortable with these concepts before starting Investigation 2.

Addition and Subtraction Properties	Multiplication and Division Properties
<p>For all real numbers a, b, and c:</p> <ul style="list-style-type: none"> ○ $a < b \Leftrightarrow a + c < b + c$ ○ $a > b \Leftrightarrow a + c > b + c$ ○ $a < b \Leftrightarrow a - c < b - c$ ○ $a > b \Leftrightarrow a - c > b - c$ 	<p>For any real numbers a, b, and c:</p> <ul style="list-style-type: none"> ○ $a < b \Leftrightarrow ac < bc$ ○ $a > b \Leftrightarrow ac > bc$ ○ $a < b \Leftrightarrow \frac{a}{c} < \frac{b}{c}$ ○ $a > b \Leftrightarrow \frac{a}{c} > \frac{b}{c}$ <p>if and only if c is positive ($c > 0$)</p>



In Investigation 1, the following scenario is introduced:

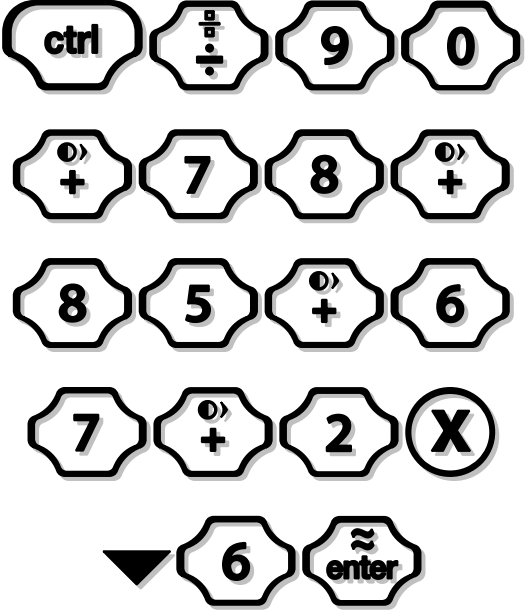






In one course, your final grade is based on the average of 6 hours of exams you take. The first four exams are one-hour exams and these are the scores.

90, 78, 85, 67

Your final exam is a two-hour test. What grade do you need to score on the final exam in order to average 85 or better in the course?

The solution to this problem is a linear inequality in one variable, x . The problem itself involves a weighted average. The solution is shown in the video without the use of a graphing calculator. However, as an alternative solution, here is a way to integrate the use of the TI-Nspire.

TI-Nspire Keystrokes	
Turn on the TI-Nspire. Press the home key then 6 to open a new document.	
You may be prompted to save and open document. After you decide, select 2 to create a Graphs and Geometry Page.	

<p>At the f1 line input this expression: $\frac{90 + 78 + 85 + 67 + 2x}{6}$</p>	
<p>At the f2 line input the number 85.</p>	
<p>Although you can't see them, the graphs are there. To view them use the Zoom Out option.</p>	
<p>Click multiple times until you can see the intersection point.</p>	
<p>Find the coordinates of the intersection point.</p>	
<p>Use the NavPad to move the pointer above each point and click on it.</p>	<p>Use  to move the pointer over the graph of f1. Press . Repeat with f2.</p>
<p>The x-coordinate of the intersection point is the minimum grade needed to get a final score of 85. These results agree with the solution shown in the video.</p>	

Practice

Use the scenario from the investigation to determine the minimum score needed on the final exam.

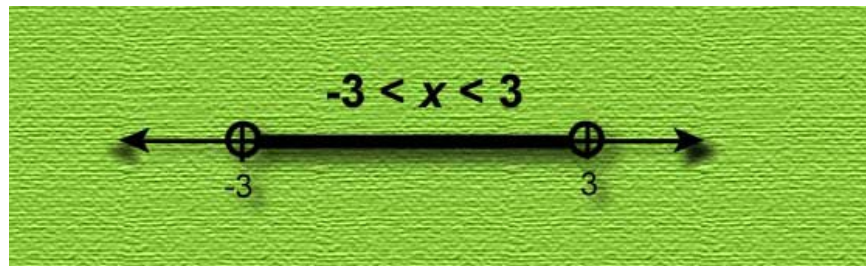
1. 92, 75, 75, 87
2. 95, 87, 88, 100
3. 25, 100, 100
4. Why is it impossible for someone who got scores of 68, 65, 100, 72 to get a grade in the course of 85?

Assessment

Compound inequalities can sometimes be written as absolute value inequalities.



$$x < -3 \text{ OR } x > 3 \rightarrow |x+3| > 6$$

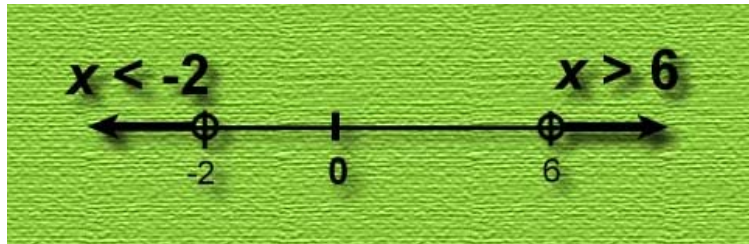


$$x < -3 \text{ AND } x > 3 \rightarrow |x| < 3$$

Find the absolute value inequalities for these compound inequalities.

- a. $x < -5$ OR $x > 5$
- b. $x > -10$ AND $x \leq 1/2$

Now see if you can find the absolute value inequality whose graph is shown below.




















Investigation 2


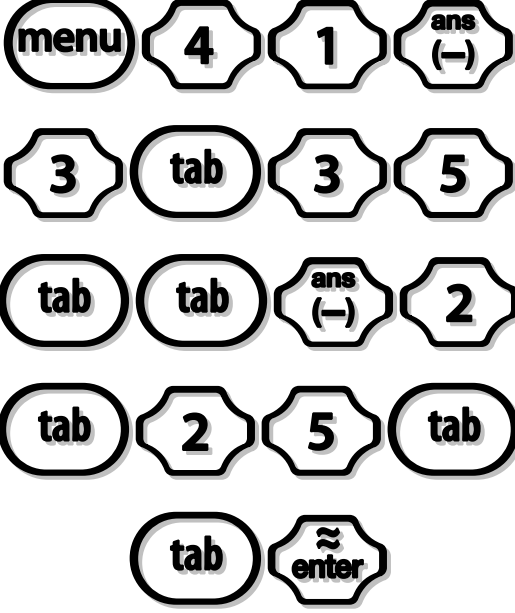





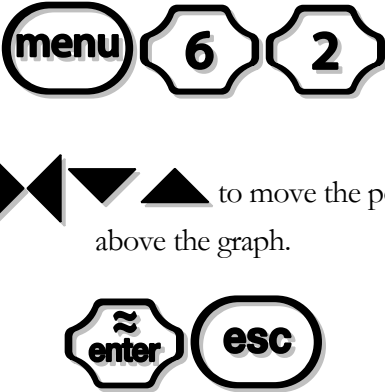

In this investigation, students solve the following problem using the TI-Nspire:










A business owner runs a yoga studio. She offers two options: \$12 per class if you buy a pass of 10 classes, and \$18 per class if you just drop in.

Open seven days a week, the total cost of running the studio—including rent, utilities, advertisement, and salaries—works out to an average of \$360 per day.

How many students does she need each day to make a profit?

TI-Nspire Keystrokes	
Turn on the Nspire.	
Press the home key followed by 6, or ctrl N to open a new document.	  OR  
You may be prompted to save and open document. After you decide, select 2 to create a Graphs and Geometry Page.	  
Input this function $-\frac{2}{3}x + 20$ Press enter to graph. Press esc to move the cursor from the entry line to the work area.	        

	
<p>To change the window, press menu, and under window select window settings. Enter the following values, using tab to move after each entry: -3, 35, -2, 25. Then click OK.</p>	
<p>Use the Nav Pad to drag the equation upward. Press esc to exit grab-and-drag mode.</p>	<p>Use  to move the pointer above the equation. Press  until  changes to . Use  to move the equation label to the top of the screen.</p>
<p>Place a movable point on the line, press menu and under Points and lines, select Point on. Move the pointer to any point on the line. When you see the words "point on" press enter to label the coordinates. Press esc to exit point on mode. Notice the blinking point and open hand. Click and hold to grab the point. Now use the Nav Pad to slide the point as you travel up and down the line. Notice how the coordinate values, or x and y</p>	 <p>Use  to move the pointer above the graph.</p>

<p>values, change accordingly. All of these points represent break-even points, where expenses equal income.</p>	<p>Press  until  changes to . Use  to move the point.</p>
<p>Graph the inequality shown.</p> $y > -\frac{2}{3}x + 20$ <p>In the function entry line, we can type in an inequality as follows: press the backspace key to erase the equal sign then press >. Press enter and watch the top half-plane become shaded. This shaded region is the infinite set of all ordered pairs that are solutions to our inequality.</p>	<p>  </p> <p> </p>

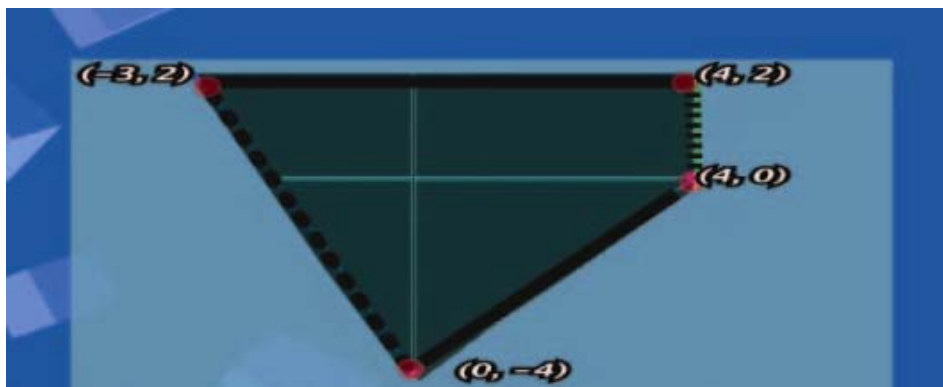
Practice

Graph the following inequalities.

1. $y \leq 2x + 1$
2. $y > -3x + 5$
3. $y \geq -0.5x + 200$

Assessment

The graphical solutions of systems of three or more linear inequalities in two variables are often polygonal in shape. Look at the following illustration.



Use the information in the graph to find the three linear inequalities.

NCTM Standards Correlation					
<p>Each of the videos in the <i>Algebra Nspirations</i> series is correlated to the NCTM Standards. This table identifies the particular process and content standards addressed by each video.</p>	Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
	<p>Process Standards</p> <p>Problem Solving</p> <p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p>				
Build new mathematical knowledge through problem solving	✓	✓	✓	✓	✓
Solve problems that arise in mathematics and in other contexts	✓	✓	✓	✓	✓
Apply and adapt a variety of appropriate strategies to solve problems	✓	✓	✓	✓	✓
Monitor and reflect on the process of mathematical problem solving					
<p>Reasoning and Proof</p> <p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p>					
Recognize reasoning and proof as fundamental aspects of mathematics					
Make and investigate mathematical conjectures	✓	✓	✓	✓	✓
Develop and evaluate mathematical arguments and proofs	✓	✓	✓	✓	✓
Select and use various types of reasoning and methods of proof					

NCTM Standards Correlation	Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
Communication					
Instructional programs from prekindergarten through grade 12 should enable all students to—					
Organize and consolidate their mathematical thinking through communication	✓	✓	✓	✓	✓
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others	✓	✓	✓	✓	✓
Analyze and evaluate the mathematical thinking and strategies of others;	✓	✓	✓	✓	✓
Use the language of mathematics to express mathematical ideas precisely.	✓	✓	✓	✓	✓
Connections					
Instructional programs from prekindergarten through grade 12 should enable all students to—					
Recognize and use connections among mathematical ideas	✓	✓	✓	✓	✓
Understand how mathematical ideas interconnect and build on one another to produce a coherent whole					
Recognize and apply mathematics in contexts outside of mathematics	✓	✓	✓	✓	✓
Representation					
Instructional programs from prekindergarten through grade 12 should enable all students to—					
Create and use representations to organize, record, and communicate mathematical ideas	✓	✓	✓	✓	✓
Select, apply, and translate among mathematical representations to solve problems	✓	✓	✓	✓	✓
Use representations to model and interpret physical, social, and mathematical phenomena	✓	✓	✓	✓	✓

NCTM Standards Correlation		Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
Content Standards						
Algebra						
Instructional programs from prekindergarten through grade 12 should enable all students to—						
Understand patterns, relations, and functions						
Grades 6–8 Expectations:						
represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;	✓	✓		✓		
relate and compare different forms of representation for a relationship;	✓	✓	✓	✓	✓	
identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.	✓	✓	✓	✓		

<h2 style="text-align: center;">NCTM Standards Correlation</h2>	Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
Grades 9–12 Expectations:					
generalize patterns using explicitly defined and recursively defined functions;					
understand relations and functions and select, convert flexibly among, and use various representations for them;				✓	
analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;	✓	✓	✓		
understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;					
understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;					
interpret representations of functions of two variables					

<h2 style="text-align: center;">NCTM Standards Correlation</h2>	Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
Grades 6–8 Expectations:					
develop an initial conceptual understanding of different uses of variables;	✓	✓	✓	✓	
explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;	✓				
use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;	✓		✓	✓	
recognize and generate equivalent forms for simple algebraic expressions and solve linear equations	✓		✓		
Grades 9–12 Expectations:					
understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;	✓	✓	✓	✓	✓
write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;	✓				✓
use symbolic algebra to represent and explain mathematical relationships;	✓	✓	✓	✓	✓
use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;					
judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.	✓	✓	✓	✓	✓

NCTM Standards Correlation		Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
Use mathematical models to represent and understand quantitative relationships						
Grades 6–8 Expectations:						
model and solve contextualized problems using various representations, such as graphs, tables, and equations.		✓	✓	✓	✓	✓
Grades 9–12 Expectations:						
In grades 9–12 all students should–						
identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;		✓	✓		✓	
use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;		✓	✓	✓	✓	✓
draw reasonable conclusions about a situation being modeled.		✓	✓	✓	✓	✓
Analyze change in various contexts						
Grades 6–8 Expectations:						
use graphs to analyze the nature of changes in quantities in linear relationships.		✓		✓		✓
Grades 9–12 Expectations:						
approximate and interpret rates of change from graphical and numerical data.		✓	✓		✓	