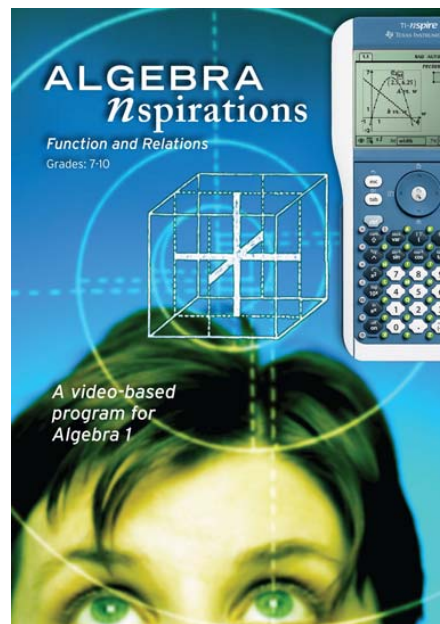




## ALGEBRA NSPIRATIONS

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### Functions and Relations



# Teacher's Guide

## Series Overview

For Algebra 1 teachers looking for a video resource that uses graphing calculators, *Algebra Nspirations* provides an ideal solution. Each program in this series focuses on a key topic in algebra and uses real-world examples to explore these topics. In addition, all the relevant calculator keystrokes for the TI-Nspire calculator are provided. In addition, Math Labs allow for hands-on exploration of these topics.

## Program Overview

Functions are relationships among quantities that change. Written and hosted by internationally acclaimed math educator Dr. Monica Neagoy, this video explores definition, vocabulary, and notation; distinguishes the concept of function from a general relation; and integrates the TI-Nspire in the dynamic exploration of the multiple representations of functions, while simultaneously connecting them to real-world contexts.

Concepts explored: functions, relations, equations, quadratic functions, linear functions, multiple representations





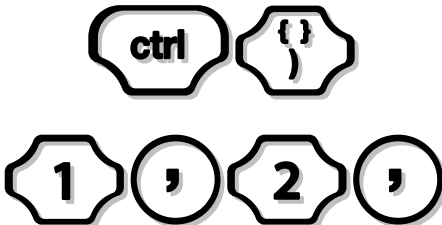
**Grades: 7-12+**

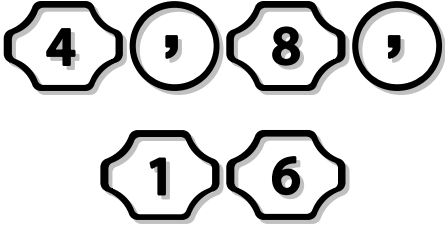
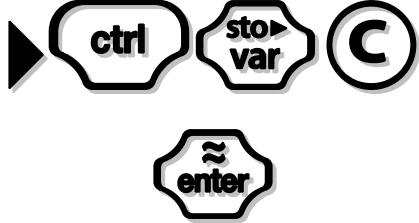
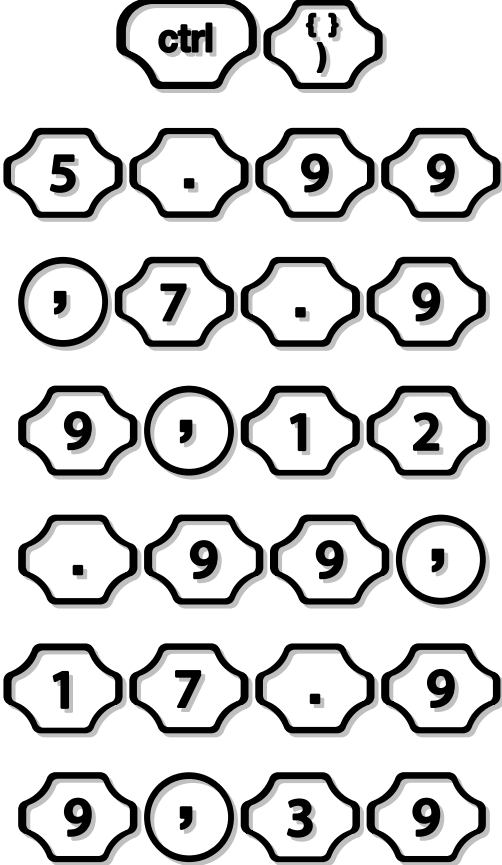
## Investigation 1


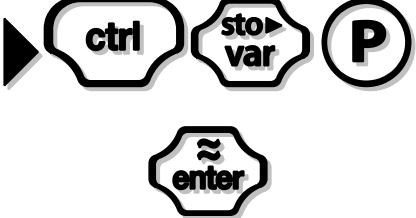

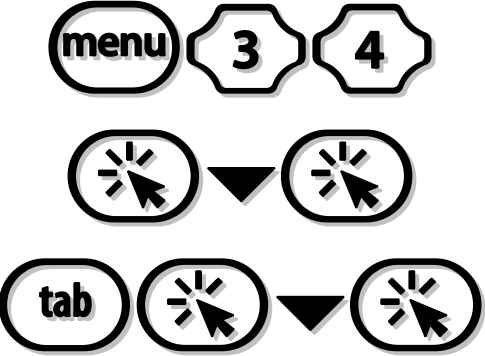
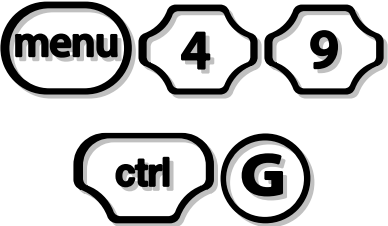
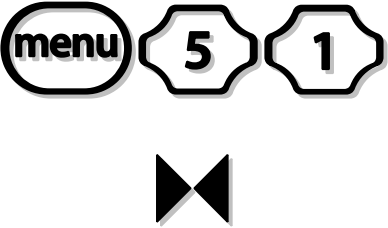
In the first investigation, students explore the relationship between two quantities: flash drive capacity and the price of the flash drive. The data is summarized in this table.

Capacity (GB)	Price (\$)
1	5.99
2	7.99
4	12.99
8	17.99
16	39.99

Students create a list and a scatterplot.

TI- <i>n</i> spire Keystrokes	
Press the home key and select 6 for a new document.	
A previous document may be open: if so, a prompt will ask if you wish to save the document. Click to choose “yes” or press tab then click to choose “no.”	 OR 
After you decide, select 1 to create a Calculator page.	
To store the domain values as a list named c, press CTRL followed by the right parenthesis symbol; then key in 1, 2, 4, 8, and 16, inserting a comma each time.	





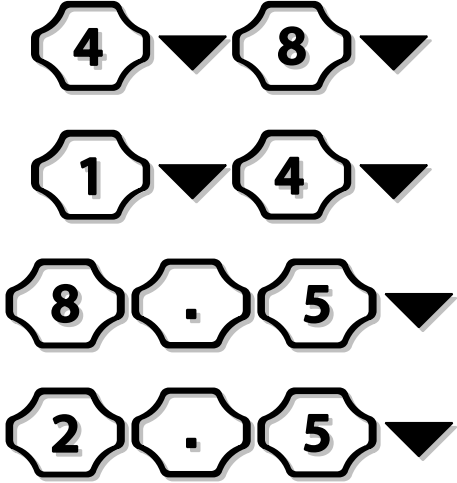
	
<p>Press the right arrow to move outside of the braces, then CTRL and VAR to store these numbers. Type the letter c, using the green letter key, then press enter. This is now labeled as our c-list.</p>	
<p>To do the same for the range values, press CTRL and the right parenthesis key, then type 5.99, 7.99, 12.99, 17.99 and 39.99. Don't forget the commas.</p>	

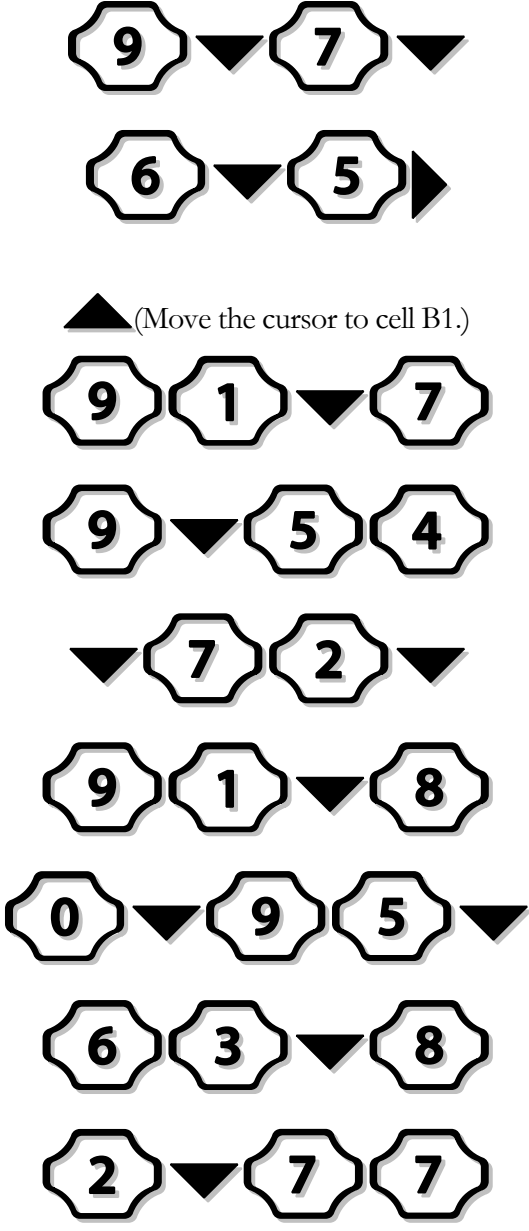

	
<p>When you're done, press the right arrow, and then ctrl var to store, the letter p for price, and then enter. This is our p-list.</p>	
<p>To graph the (c, p) points, access the Graphs &amp; Geometry application by pressing Home, followed by 2.</p>	
<p>Press Menu and under Graph Type, select Scatter Plot. The domain or x-box is highlighted. Click to see your choices, select c, and press enter. Tab over to the range or y-box, click again but this time select p. Press enter.</p>	
<p>To create an appropriate window for this graph, press Menu and under Window select Zoom Data. Press ctrl G to hide the entry line along the bottom.</p>	
<p>To see the points' coordinates, press Menu one last time and under trace select graph trace. Use the Navigation Pad or Nav Pad to move from one point to the next.</p>	

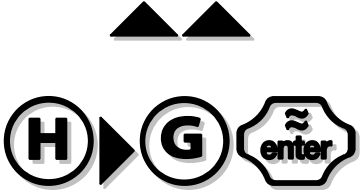

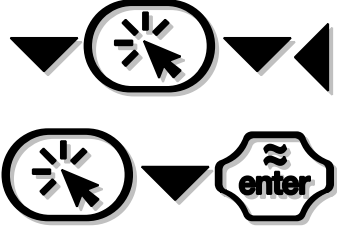
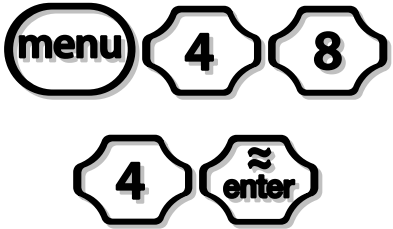
In this part of Investigation 1, students explore the relationship between the number of hours studied and the grade on a test. These are the data points:

(4, 91) (8, 79) (1, 54) (4, 72)  
 (8.5, 91) (2.5, 80) (9, 95)  
 (7, 63) (6, 82) (5, 77)

Students use a spreadsheet and scatterplot to analyze the data.

TI- <i>n</i> spire Keystrokes	
Press the home key and select 6 for a new document.	
A previous document may be open: if so, a prompt will ask if you wish to save the document. Click to choose “yes” or press tab then click to choose “no.”	 OR 
Select 3 to create a Lists and Spreadsheet page.	
Input the following data into columns A and B of the spreadsheet.  (4, 91) (8, 79) (1, 54) (4, 72) (8.5, 91) (2.5, 80) (9, 95) (7, 63) (6, 82) (5, 77)	

	
<p>Scroll up column A and above the header, type h. Click or enter, then go to the same cell in Column B and type g. Press enter.</p>	 <p>(Move the cursor to cell A1.)</p>

	
<p>To graph the scatter plot, you'll need to open a new page. So press the Home key and select 5 for a Data and Statistics Page.</p>	
<p>To make some sense of this scatter plot, use the down arrow to move the pointer to the x-axis. A box appears with "click to add variable" inside. Click, then select h for the x-axis variable; press enter. Next, use the left arrow to move the pointer to the y-axis until the same box appears. Click, then select g for the y-axis variable, then press enter.</p>	
<p>Press Menu and under Analyze select Plot Value. Input 4 for V1, then press enter. The vertical line <math>x = 4</math> passes through two different points. This means that two different students each studied the same number of hours—four, but earned two different grades. Checking the ordered pairs, we find (4, 72) and (4, 91).</p>	

Practice

Use a calculator window and scatterplot to analyze these data sets.

1.

$x$	$y$
1	-1
3	15
3	-2
5	7
7	20
12	15
13	20
13	21
21	12

2.

$x$	$y$
-5	0
-2	5
-1	17
0	0
2	9
3	7
6	-15
10	-60
12	9

2.

Use a spreadsheet window and scatterplot to analyze these data sets.

3.

$x$	$y$
1	1
2	2
3	3
4	4
4	5
5	5
6	6
7	7
8	8

4.

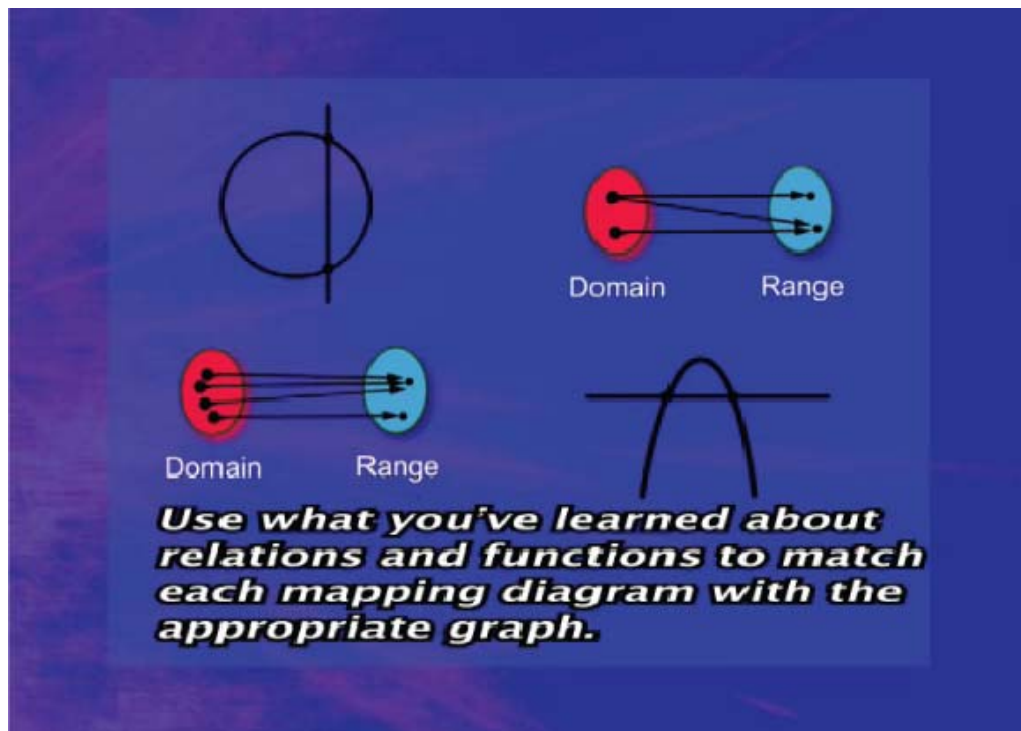
$x$	$y$
1	25
2	20
3	15
4	10
5	5
6	0
7	-5
8	-10
9	-15

4. Identify which of the previous data sets are functions.

## Assessment

The *Try This* sections of the video, also known as the Math Labs, encourage students to think critically about the concepts they have learned.

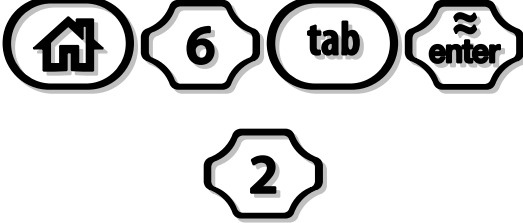
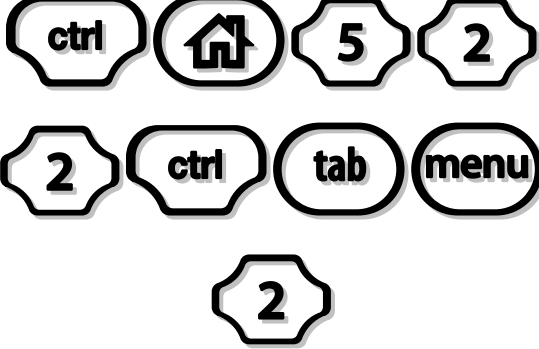
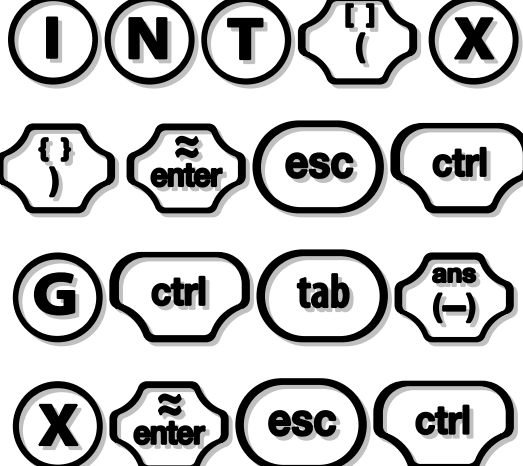
In Math Lab 1 students analyze mapping diagrams to identify functions and relations. Students are asked to match the graph with the mapping diagram.


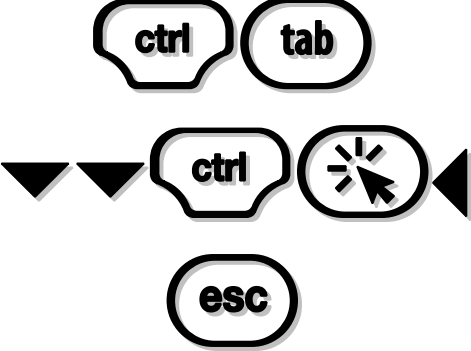






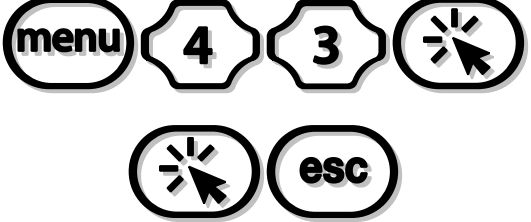
***Use what you've learned about relations and functions to match each mapping diagram with the appropriate graph.***

## Investigation 2

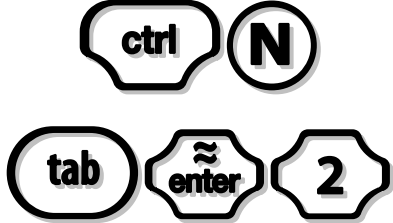
In this investigation, students graph and analyze graphs of linear, quadratic, rational, and discrete functions. Then students link the measurements of a geometric construction to a graph and analyze the resulting function graph.

















TI- <i>n</i> spire Keystrokes	
<p>Press the home key, then 6 to open a new document.            Decide if you wish to save the open document, then press 2 for a Graphs and Geometry Page.</p>	
<p>Press ctrl ... home, and under page layout choose select layout and pick layout 2, the vertical split screen. Press ctrl tab to move to the right. Next, press menu and again select 2.</p>	
<p>We'll graph two functions on each of these Cartesian planes. The cursor is blinking by f1: key in the letters i, n, t, then the left paren, x, and finally the right paren. For f2 type in - x. Make sure to use the white key for the negative sign. Press esc to move to the work area and ctrl G to hide the entry line.</p>	


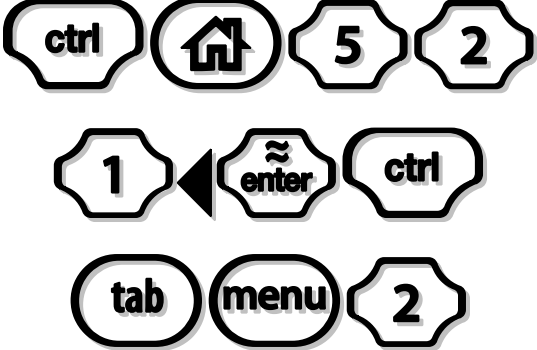
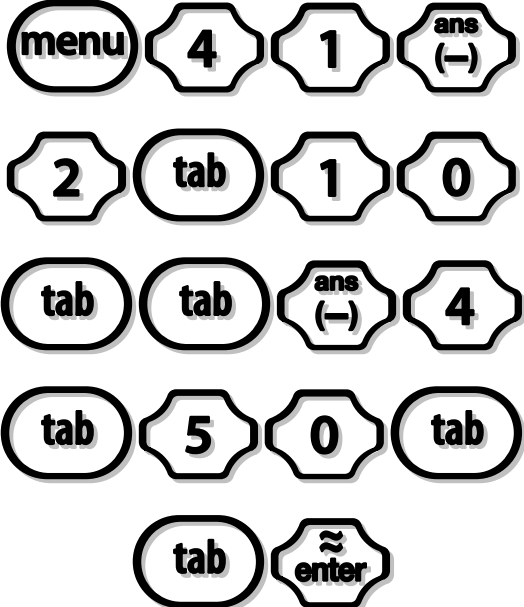
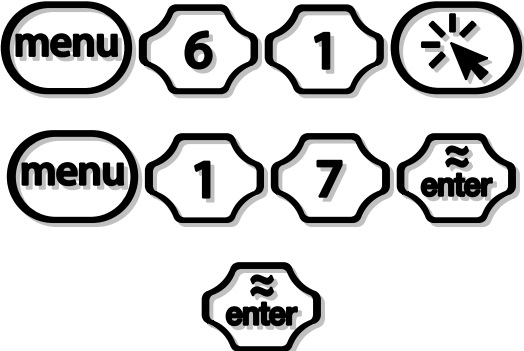

	
<p>To zoom in and see better, use the Nav Pad to move the pointer the first tick mark on the x axis, to the left of the origin. When it becomes an open hand, press ctrl and click to grab this point. Use the left arrow to drag it all the way to the left edge of the monitor. Press esc to exit grab and drag. You can now see unit 1 on the x axis.</p>	
<p>The first function is called the integer part function: its graph is like a staircase and often called a step function: it assigns the integer part of <math>x</math> to any real number <math>x</math>.        The second is a linear function with slope <math>-1</math>. Negative slopes yield descending graphs from left to right. Linear functions have straight-line graphs as you may have guessed.        A vertical line test from left to right yields exactly one intersection point—on each graph—for every value of <math>x</math>. That confirms that we have two functions and it also tells us that in both cases the domain and range are all real numbers.</p>	
<p>Press Ctrl tab to move to the left</p>	
<p>Type in x-squared for f3. For x-squared, make sure to press the green x key followed by the x-squared key.</p>	
<p>For our fourth function, enter 1 divided by x for the reciprocal of x. Then press esc.</p>	

	
<p>To zoom in this time, press menu and under Window select Zoom In. A center box appears. Press click twice then esc.</p>	
<p>X-squared is a quadratic function and the shape of all quadratic function graphs is called a parabola. This parabola lives in quadrants 1 and 2, where all y-values are positive. The two northbound branches continue upward to infinity.</p> <p>A vertical line test yields exactly one intersection point for every value of x.</p> <p><math>1/x</math> is called a rational function and its graph is called a rectangular hyperbola. This hyperbola lives in quadrant 1—where y is positive, and quadrant 3—where y is negative. It has four infinite branches and nowhere does the graph ever touch the x or y axis: So, x and y are never zero.</p> <p>A vertical line test yields exactly one intersection point for every value of x except zero.</p>	

In this part of the investigation students construct a circle and graph the radius versus the perimeter of the circle, for different values of each.


TI- <i>n</i> spire Keystrokes	
<p>Press Ctrl N for a new document. Select 2 for graphs and geometry.</p>	




<p>Press menu and under view select plane geometry view. We're going to construct a circle.</p>	
<p>Press menu and under shape, select circle. Move the point toward the left of the monitor and click to place the Circle's center.</p>	 <p>(Use    to move to the left part of the screen.)</p>
<p>Use the nav pad to create a circle then click again. Now we measure the radius: press menu and under measurement, pick length.</p>	 
<p>Move the cursor to the center, click, then drag it with the down arrow until you reach a point on the circumference. Click twice and the radius length appears.</p>	 
<p>Next we measure the perimeter: press menu and under measurement, pick length again. With the Nav Pad move to a point at the top of the circle and click twice. Now the perimeter appears</p>	  
<p>To name this variable press the var key and select store var. Type the letter p for perimeter, then press enter. Do the same for the radius: hover over it and likewise press var. Choose store var again and this time type r for radius, then press enter. You've now labeled both variables.</p>	 <p>Use    to move the pointer above the measurement of the radius.</p>


	
<p>We're ready to split the screen, so press ctrl home and under page layout select custom split. With the Nav Pad, drag the partition to the left, making the left screen about one third the screen's width, then press enter. Press ctrl tab to move to the right, menu to make a selection, and 2 for a graph window. You now have a plane geometry page on the left and an analytical geometry page on the right.</p>	
<p>Since radii and perimeters or—circumferences in this case—are positive, let's change the settings: press menu and under window select window settings. Enter -2, 10, -4, and 50. Click OK.</p>	
<p>To place a point, press menu and under points and lines pick point. Click to plot a point. Next press menu again and under actions select coordinates and equations. Press Enter twice to label the coordinates.</p>	
<p>We now want to connect the x-value with the radius and the y-value with the perimeter. To do this, hover over the x-</p>	<p>Use  to move the pointer</p>

coordinate of the point and press var. This time select link to and then the letter r for radius. Press enter. Do the same for the y-coordinate: press var, link to the letter p for perimeter, then press enter.


above the x-coordinate.










Use    to move the pointer above the y-coordinate.



Finally, switch back over to the left screen by pressing ctrl tab. Move the pointer to a point on the circle. When the word circle appears, click and hold to grab the point. Use the Nav Pad to change the size of the circle. Notice what happens to the point on the other screen.



Use    to move the pointer above the circle. Click and hold  until  changes to .



## Practice

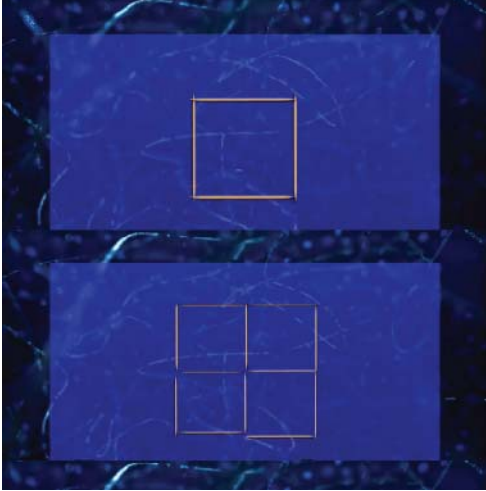
Use the results of the circle activity to answer the questions below.

1. Describe what happens to the point on the right side of the screen.
2. The relationship between  $r$  and  $p$  is a function. Which is the independent variable and which is the dependent variable?
3. Find the equation of the line that is described by the movement of the point on the right side of the screen.

## Assessment

The *Try This* sections of the video, also known as the Math Labs, encourage students to think critically about the concepts they have learned.

In Math Lab 2 students gather data based on a toothpick activity. Students will need a box of toothpicks for this activity. Students construct square shapes and record the number of toothpicks on each side of the square versus the total number of toothpicks used. The first two entries are shown below. Students record their data in a chart.

	<table border="1"> <thead> <tr> <th><i>Number of toothpicks on a side</i></th> <th><i>Total number of toothpicks</i></th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table>	<i>Number of toothpicks on a side</i>	<i>Total number of toothpicks</i>										
<i>Number of toothpicks on a side</i>	<i>Total number of toothpicks</i>												

Once students generate a sufficient amount of data, they should graph it and describe the functional relationship between the variables, as well as the shape of the graph.

<p style="text-align: center;"><b>NCTM Standards Correlation</b></p> <p>Each of the videos in the <i>Algebra Nspirations</i> series is correlated to the NCTM Standards. This table identifies the particular process and content standards addressed by each video.</p>	Linear Functions, Equations, and Graphs	Quadratic Functions, Equations, and Graphs	Variables, Equations, and Functions	Functions and Relations	Inequalities
<b>Process Standards</b>					
<b>Problem Solving</b>					
Instructional programs from prekindergarten through grade 12 should enable all students to—					
Build new mathematical knowledge through problem solving					
Solve problems that arise in mathematics and in other contexts					
Apply and adapt a variety of appropriate strategies to solve problems					
Monitor and reflect on the process of mathematical problem solving					
<b>Reasoning and Proof</b>					
Instructional programs from prekindergarten through grade 12 should enable all students to—					
Recognize reasoning and proof as fundamental aspects of mathematics					
Make and investigate mathematical conjectures					
Develop and evaluate mathematical arguments and proofs					
Select and use various types of reasoning and methods of proof					

<b>Communication</b>	
<b>Instructional programs from prekindergarten through grade 12 should enable all students to—</b>	
Organize and consolidate their mathematical thinking through communication	■ ■ ■ ■ ■
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others	■ ■ ■ ■ ■
Analyze and evaluate the mathematical thinking and strategies of others;	■ ■ ■ ■ ■
Use the language of mathematics to express mathematical ideas precisely.	■ ■ ■ ■ ■
<b>Connections</b>	
<b>Instructional programs from prekindergarten through grade 12 should enable all students to—</b>	
Recognize and use connections among mathematical ideas	■ ■ ■ ■ ■
Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	■ ■ ■ ■ ■
Recognize and apply mathematics in contexts outside of mathematics	■ ■ ■ ■ ■
<b>Representation</b>	
<b>Instructional programs from prekindergarten through grade 12 should enable all students to—</b>	
Create and use representations to organize, record, and communicate mathematical ideas	■ ■ ■ ■ ■
Select, apply, and translate among mathematical representations to solve problems	■ ■ ■ ■ ■
Use representations to model and interpret physical, social, and mathematical phenomena	■ ■ ■ ■ ■
<b>Content Standards</b>	
<b>Algebra</b>	
<b>Instructional programs from prekindergarten through grade 12 should enable all students to—</b>	
<b>Understand patterns, relations, and functions</b>	
<b>Grades 6–8 Expectations:</b>	
represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;	■ ■ ■ ■ ■
relate and compare different forms of representation for a relationship;	■ ■ ■ ■ ■
identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.	■ ■ ■ ■ ■
<b>Grades 9–12 Expectations:</b>	
generalize patterns using explicitly defined and recursively defined functions;	■ ■ ■ ■ ■

understand relations and functions and select, convert flexibly among, and use various representations for them;					
analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;					
understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;					
understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;					
interpret representations of functions of two variables					
<b>Represent and analyze mathematical situations and structures using algebraic symbols</b>					
<b>Grades 6–8 Expectations:</b>					
develop an initial conceptual understanding of different uses of variables;					
explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;					
use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;					
recognize and generate equivalent forms for simple algebraic expressions and solve linear equations					
<b>Grades 9–12 Expectations:</b>					
understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;					
write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;					
use symbolic algebra to represent and explain mathematical relationships;					
use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;					
judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.					
<b>Use mathematical models to represent and understand quantitative relationships</b>					
<b>Grades 6–8 Expectations:</b>					

model and solve contextualized problems using various representations, such as graphs, tables, and equations.	■ ■ ■ ■ ■
<b>Grades 9–12 Expectations:</b>	
In grades 9–12 all students should–	
identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;	■ ■ ■ ■ ■
use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;	■ ■ ■ ■ ■
draw reasonable conclusions about a situation being modeled.	■ ■ ■ ■ ■
<b>Analyze change in various contexts</b>	
<b>Grades 6–8 Expectations:</b>	
use graphs to analyze the nature of changes in quantities in linear relationships.	■ ■ ■ ■ ■
<b>Grades 9–12 Expectations:</b>	
approximate and interpret rates of change from graphical and numerical data.	■ ■ ■ ■ ■