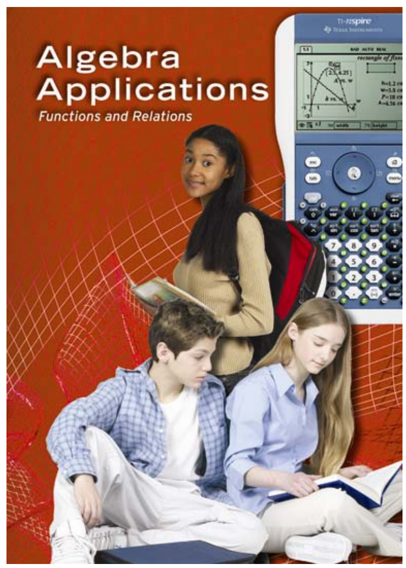
ALGEBRA APPLICATIONS

Functions and Relations



Teacher's Guide

Series Overview

The *Algebra Applications* series brings real-world, relevant applications of algebra to today's classroom. This series also integrates technology through the use of the Texas Instruments TI-Nspire graphing calculator. The key features of this series include:

- Guided applications that are interdisciplinary and can be done as an in-class group activity.
- All keystrokes are clearly shown.
- Dynamic footage and animations bring math to life.
- Math concepts are developed clearly, making this series an ideal supplement to an Algebra 1 or Algebra 2 class.

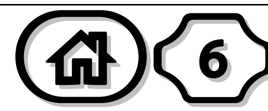
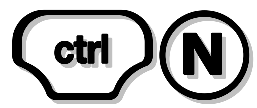
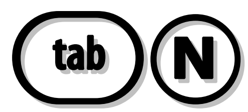
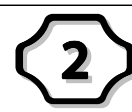
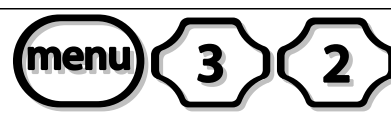
Program Overview

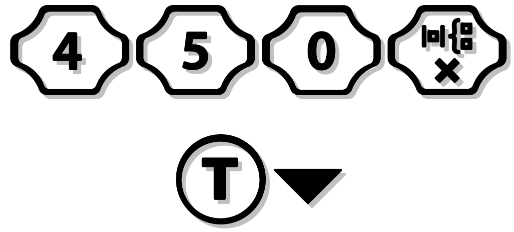
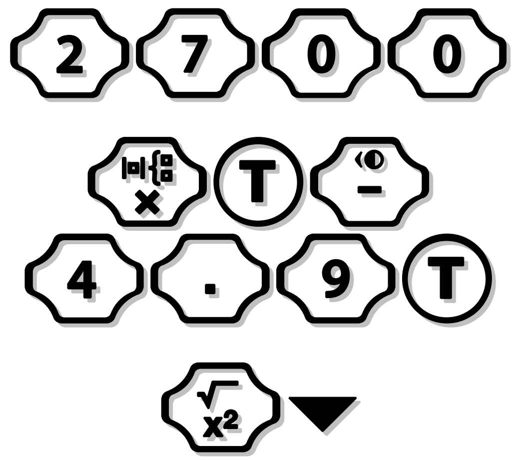
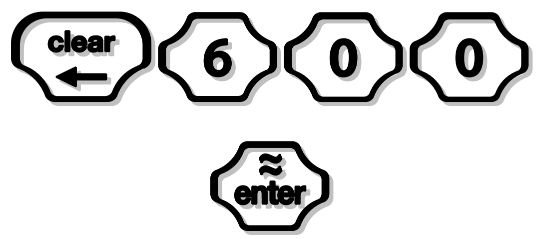
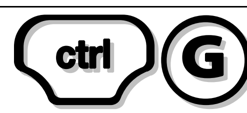
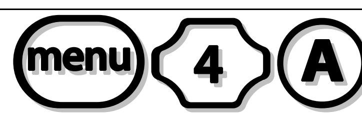
In this episode of *Algebra Applications* a simulated future journey to Mars offers a way of exploring conic sections, and their underlying functions. Students use the Graphs and Geometry features of the TI-Nspire. The program is segmented into the particular trajectories of each part of the space journey:

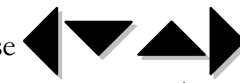
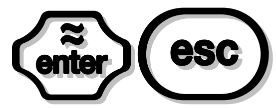
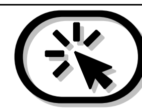
- **Parabolic paths.** The liftoff and pre-orbital path of the rock is described by a parabola. Students explore the properties of parabolas from the standpoint of the parametric equations that describe the horizontal and vertical directions of motion of the rocket.
- **Circular paths.** The path of a rocket orbiting the Earth can be modeled with the equation of a circle. Students explore the quadratic relation and the parametric equations that can be used to model the path of a spacecraft orbiting Earth.
- **Elliptical paths.** The planets orbiting the sun follow elliptical paths. In fact, the trajectory of a spacecraft traveling to Mars would also be elliptical. Students explore these various ellipses.

Application 1: Parabolic Paths

In this application, students analyze the parabolic motion of the rocket at liftoff and through the first part of the journey that takes it to the International Space Station. Students use the parametric equations capabilities of the TI-Nspire. .

TI-*n*spire Keystrokes	
Turn on the *n*spire.	
Press the home key followed by 6, or **ctrl N** to open a new document.	 OR 
A previous document may be open: if so, a prompt will ask if you wish to save the document. Click to choose "yes" or press tab then click to choose "no."	 OR 
Press 2 to create a Graphs and Geometry Window.	
To activate the parametric equation capability press Menu, 3, 2. Notice that unlike the function graph capability, two equations are shown, one for x and one for y. With a parametric equation both x and y are dependent on another variable, in this case t.	

Input the following parametric equations: $x(t) = 450t$ $y(t) = 2700t - 4.9t^2$ Input the equation for x(t) and press the down arrow to move to the y(t) field. Then input the equation for y(t) and press the down arrow to move to the field for setting the range for t. We want to graph for the first ten minutes of the flight, or 600 seconds. Change the value to the right of t to 600. Press the Clear key to remove the previous value and replace it with 600. Then press	  
Press Ctrl and G to remove the equation entry area.	
To see all of the graph, press Menu, 4, and A to choose the Zoom fit option. Now you can see the whole graph, and you can now see that this parametric graph is a parabola. Notice that this graph passes the vertical line test, so it is a function (But not all parabolas are functions.)	

Place a point on the graph and follow its trajectory. Press Menu, 6, and 2 to activate the point tool. Use the NavPad to move the pointer on the parabola. Press Enter. Then press Esc. Notice that a point is on the parabola with the coordinates displayed.	 (Use  to move the pointer over the parabola.) 
Now make sure the pointer is above the point. The pointer should change to an open hand. Press and hold the Click key till the pointer changes to a closed hand. Use the right arrow to move the point along the graph. Notice how the points change. When the point reaches a y-coordinate of around 370,000 meters, or 370 km, it reaches the altitude of the International Space Station, which is the first stop on the trip to Mars.	 (Click and hold until  changes to this  .) 

Assessment

Determine if the parabola is a function.

1. $f(t)=\begin{cases} x(t)=5t \\ y(t)=-t^2 \end{cases}$

2. $f(t)=\begin{cases} x(t)=-5t^2 \\ y(t)=-t \end{cases}$

3. $f(t)=\begin{cases} x(t)=-t \\ y(t)=-\frac{1}{2}t^2 \end{cases}$

4. $f(t)=\begin{cases} x(t)=\sqrt{5}t \\ y(t)=\sqrt{3}t^2 \end{cases}$

5. With a parametric equation, both *x* and *y* are a function of the variable *t*. You can rewrite these functions so that *y* is a function of *x*. Fill in the missing steps to show how to convert from the parametric form to the x-y form.

$$f(t)=\begin{cases} x(t)=t+4 \\ y(t)=t^2+t \end{cases}$$

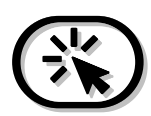
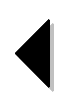
$$\begin{aligned} x(t)&=t+4 \\ t&=x-[\_\_] \end{aligned}$$

$$\begin{aligned} y&=([\_\_\_])^2+(x-4) \\ &=x^2-8x+16+[\_\_\_] \\ &=x^2+[\_\_\_\_\_] \end{aligned}$$

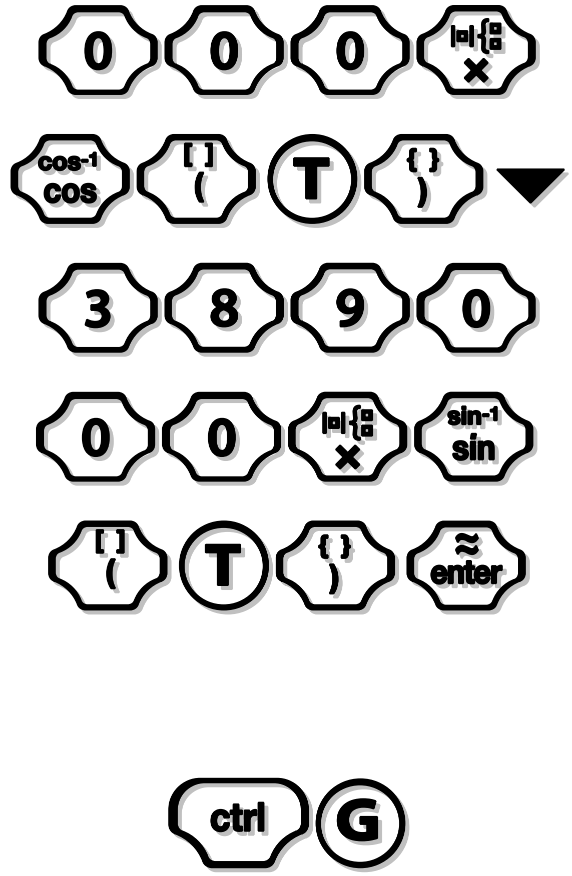
Check your work by graphing the parametric equations and the *x-y* function.

Application 2: Circular Paths

In this application, students investigate the circular orbit of a rocket around the Earth. The standard form of the equation and the parametric form are both investigated. Note: This activity assumes that students have completed the parabola activity from Application 1.

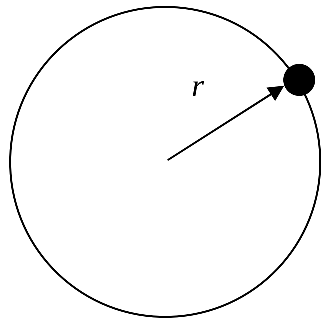
TI-*n*spire Keystrokes	
We want to create a circle whose center is at the origin and that intersects the point on the parabola that represents where the Orion docks with the International Space Station.	
Before constructing the circle, change the Window settings of the graph window. Use the NavPad to move the pointer near the origin. Press and hold the Click key till the pointer changes to a closed hand. Then use the NavPad to move the origin to the middle of the screen. Press Esc when you are done.	
Now, you want to rescale the graph window so that the circle that you are about to draw will fit. Use the NavPad to move to the horizontal axis. Move the pointer so that it is above one of the tic marks. You will see an open hand. As before, press and hold the Click key till it changes to a closed hand.	(Use ◀▼▲▶ to move the pointer over the horizontal axis.)  (Click and hold until [open hand] changes to [closed hand].)
Press the left arrow key and watch how the size of the parabola shrinks, as the graph window	

rescales.	
You are now ready to construct the circle. Press Menu, 8, and 1 to activate the circle tool.	menu 8 1
Use the NavPad to move the pointer to the origin. Press Enter. This defines the center of the circle.	(Use ◀ ▼ ▲ ▶ to move the pointer over the origin.) enter
Now use the NavPad to so that the pointer is on the point on the parabola. Press Enter. You should now see the circle.	(Use ◀ ▼ ▲ ▶ to move the pointer over the point on the parabola.) enter
Because of the way the graph was scaled your circular orbit probably looks like an ellipse. To make it look like a circle move press Menu, 4, and B to select Zoom Square. The orbit should now look like a circle.	menu 4 B
The equation of this circle is found by pressing Menu, 1, and 7.	menu 1 7
Use the NavPad so that the pointer hovers over the circle. Press Enter once, then use the down arrow to position the label for the equation. Press Enter again.	(Use ◀ ▼ ▲ ▶ to move the pointer over the circle.) enter ▼ enter
A circle can be written as a pair of parametric equations. Press Tab to bring back equation	tab 3 8 9

entry area. Input these equations: x(t) = 389,000*cos(t) y(t) = 389,000*sin(t) Press Enter. Then press Ctrl G The equations for x and y are trigonometric functions, and notice that the parametric circle overlaps the first circle. At any point in its orbit, the coordinates of the spacecraft are as shown.	

Assessment

One way to think of parametric equations is as coordinates on a graph. The parametric form of a circle has these coordinates at any point on the circle.



$(r*\cos(t), r*\sin(t))$

In the graph, *r* represents the radius of the circle.

1. Graph the following parametric circles.

   a. A circle with radius 20

   b. A circle with x-coordinates 15*cos(t)

   c. A circle with y-coordinates 4*sin(t)

2. The equation of a circle in x-y form is this:

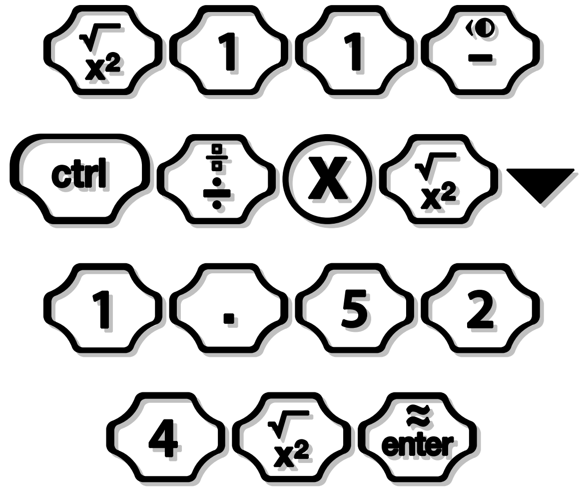
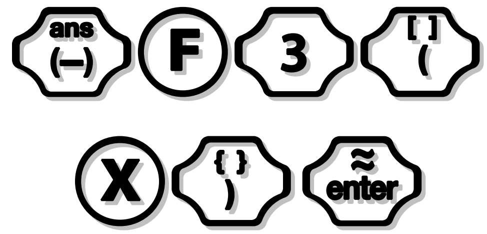
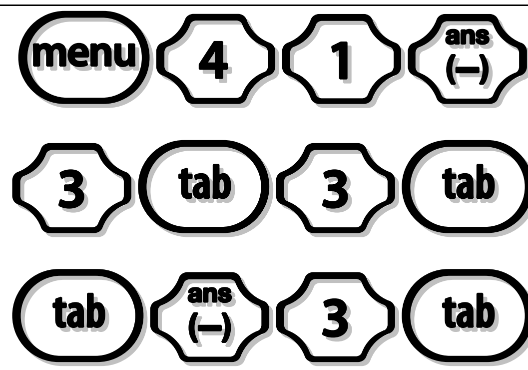
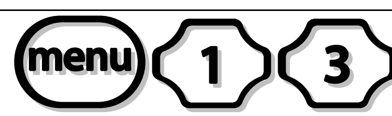
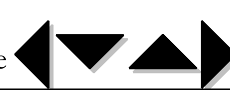
$$x^2 + y^2 = r^2$$

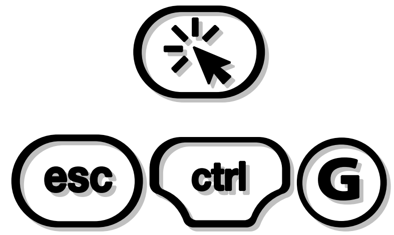
   Give three reasons why this is not a function.

Application 3: Elliptical Paths

In this application, students investigate elliptical orbits. Note: This activity can be done independently of the previous two.

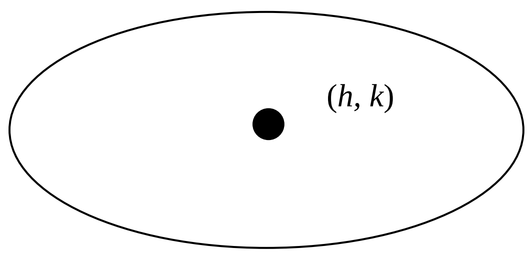
TI-*n*spire Keystrokes	
Press Home and 6.	[Home] [6]
You may need to close a previous document.	[tab] [enter]
Press 2 to create a Graphs and Geometry Window.	[2]
Input the first function for the orbit of Earth. Press Enter to graph it. This is the top half of the ellipse.	[2] [.] [9] [9] [9] [8] [6] [×] [ctrl] [√ x²] [1] [−] [X] [x²] [enter]
To graph the bottom half simply graph the negative version of f1(x). Press Enter.	[(−)] [F] [1] [(] [X] [)] [enter]
Now input the first function for the orbit of Mars. Press Enter to graph it.	[1] [.] [5] [1] [7] [4] [×] [ctrl]

To graph the other half of the ellipse, input the negative version of f3(x). Press Enter.	 
Your graph may look like two concentric circles. To get a more accurate view, change the window settings. Press Menu, 4, and 1. Change the range of x and y values to -3 and +3. Click OK to accept the changes. Now your orbits should look like ellipses. The outer ellipse is the orbit of Mars and the inner ellipse is the orbit of Earth.	
To clean up the screen press Menu, 1, and 3 to activate the hide/show option. Use the NavPad to Click on each of the equations to hide them. Press Esc and then Ctrl G.	 (Use  to move the pointer over

	each equation you wish to hide.) 

Assessment

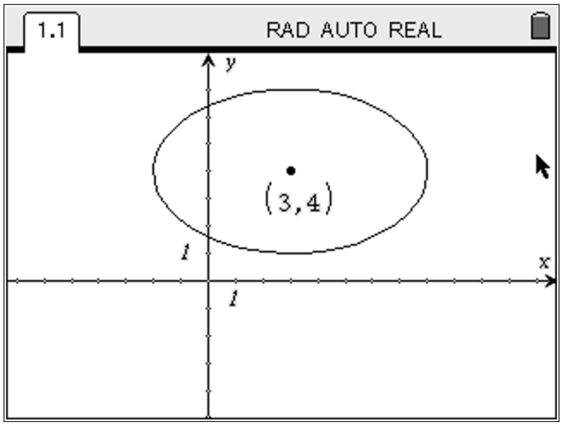
The equation of an ellipse not centered at the origin is shown below.

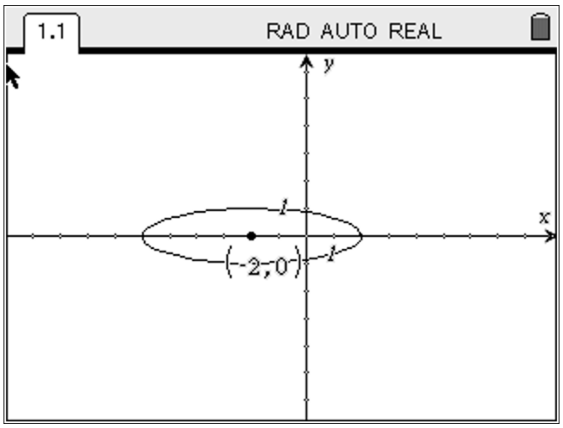


$$\frac{(x-h)^2}{a^2}+\frac{(x-k)^2}{b^2}=1$$

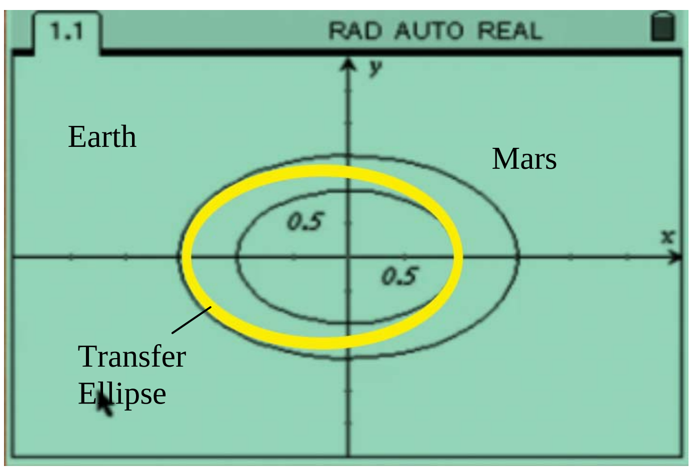
Find the equation of each ellipse, given the following conditions.

1. h= 1, k = 3.
2. h=-2, k=0

3. 

4. 

The voyage from Earth to Mars involves a transfer ellipse, where a new ellipse is formed from the elliptical paths of Earth and Mars.



5. Use the data from the video to find the equation of the transfer ellipse.

$$\frac{(x-h)^2}{a^2}+\frac{(x-k)^2}{b^2}=1$$